Topic: Intermediate value theorem without an interval

Question: The Intermediate Value Theorem can be used to prove a real root exists on a function if no interval is given.

Answer choices:

A True

B False

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Solution: A

The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is continuous on the interval, a point c exists on the interval where $f(c) = k$.

$$f(a) < k < f(b) \text{ and } a < c < b$$

If there is no interval given, but the function has restrictions on its range, the IVT can be used to prove that a real root exists.

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Topic: Intermediate value theorem without an interval

Question: There are no real roots for the function.

$$f(x) = \sin(x)$$

Answer choices:

A True

B False

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Solution: B

The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is continuous on the interval, a point c exists on the interval where $f(c) = k$.

$$f(a) < k < f(b) \text{ and } a < c < b$$

If there is no interval given, but the function has restrictions on its range, the IVT can be used to prove that a real root exists.

To check for real roots, we need to know a range. There is no given interval, but the sine function is limited to the range $[-1,1]$, so we can use that and apply the IVT.

Remember a real root occurs where the function crosses the x-axis, or the line $y = 0$. Because the range exists below and above 0, the function must cross the x-axis at least once. This means the function has at least one real root.

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Topic: Intermediate value theorem without an interval

Question: There is at least one real root for the function.

$$f(x) = \tan^{-1}(x)$$

Answer choices:

A True

B False

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Solution: A

The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is continuous on the interval, a point c exists on the interval where $f(c) = k$.

$$f(a) < k < f(b) \text{ and } a < c < b$$

If there is no interval given, but the function has restrictions on its range, the IVT can be used to prove that a real root exists.

To check for real roots, we need to know a range. There is no given interval, but the inverse tangent function is limited to the range

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

so we can use that and apply the IVT.

Remember a real root occurs where the function crosses the x-axis, or the line $y = 0$. Because the range exists below and above 0, the function must cross the x-axis at least once. This means the function has at least one real root.

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